Spatial threshold in amplifying random media

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We study experimentally as well as numerically the transport and generation of light in multiple scattering media with optical gain. By imaging the spatial distribution of light escaping from the side of the sample, the propagation depth is analyzed. Far below and far above random laser threshold, the spatial profile of emission light is independent of pump intensity, while around threshold, the spatial distribution of emission light changes profoundly. The experimental results are explained by interpreting the numerical solutions to a set of coupled time-dependent diffusion equations on a nonuniform spatial grid. Our studies provide a new and easily accessible method for observing the random laser threshold. © 2010 Optical Society of America

Understanding the transport of light inside multiple scattering media is of paramount importance for both life scientists [1] and physicists [2]. Recently, the study of light sources inside disordered media has attracted much interest [3,4]. In random lasers, sources, multiple scattering, and optical gain are combined. The majority of experimental random laser studies have focused on spectral analysis [5–7]. The spatial distribution of random laser light inside the scattering medium is largely uncharted territory. New side imaging techniques enabled the determination of important transport parameters in passive random media, such as the transport mean free path and the extrapolation length [8], and were essential in explaining the directionality of lasing in weakly scattering active media [9].

In this Letter, we exploit these new techniques to study the effect of gain on the spatial profile of emission light inside scattering samples. The results are shown to be in quantitative agreement with diffusion theory.

Samples were fabricated by suspending TiO₂ particles (R900, DuPont, median particle size 410 nm, volume fraction 1%) in an acidic solution of 1 mM Rhodamine 640 P in methanol. The experimental cell was a quartz cuvette (Hellma, inner size 10 mm × 10 mm × 45 mm, wall thickness 1.25 mm). Experiments were performed within 1 h after treatment in an ultrasonic bath, before significant sedimentation of TiO₂ particles occurred. The transport mean free path was determined via enhanced backscattering [10] to be 5 ± 2.5 μm at λ = 600 nm.

In Fig. 1 the experimental apparatus is schematically shown. Excitation light generated by an optical parametric oscillator (Opolette, 20 Hz, 5 ns, λ = 563 nm) is focused onto the sample by a microscope objective (Leitz, 0.25 NA) to a spot with a diffuse fluorescence FWHM diameter of 60 ± 4 μm. The emission collected in reflection is spectrally analyzed using a spectrograph (Oriel MS-257, resolution 0.5 nm) connected to an EMCCD camera (Hamamatsu, C-9100). We extended the conventional random laser setup with the sample side imaging technique shown by the lower arm in Fig. 1. An aspherical lens (f-number = 1.5) collects the light exiting the sample from the side and another lens images the sample surface onto a CCD camera (Mightex). Color filters were used in both detection paths to remove excitation wavelengths. The sample was placed on a translation stage in order to set the distance between the focus spot and the edge of the sample to 30 μm. The aspherical lens was also placed on a translation stage to ensure the sample surface remained in focus on the CCD camera.

The photoluminescent emission spectrum centered at 610 nm was measured for various values of the pump fluence. Figures 2(a) and 2(b) show the peak spectral radiance and the width of the output spectrum versus the averaged pump power. The peak spectral radiance and the spectral width show a clear threshold around 20 μW.

The side imaging technique makes it possible to study the propagation of light in active random media directly. Figure 3(a) shows the 50% intensity contour of the spatial profile below (0.14 μW) and above threshold (465 μW). The spatial profile of the emission light expands around threshold in both the y and z directions. The maximum of the emission profile starts to lie deeper within the sample around threshold.

To explain the experimental results, we model the transport of light by three coupled differential equations [11] in a cylindrical system. The three equations describe the diffusion of laser photon density W₁(z, r, t), pump photon density Wₚ(z, r, t), and the density of molecules in the upper laser level n₁(z, r, t):

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Schematic top view of the experimental apparatus. Green, pump beam; red, emission light. The orange bars represent color filters.
\[
\frac{\partial W}{\partial t} = D \nabla^2 W + (\sigma_e c n_1 - \sigma_r c n_0) W + \frac{\beta}{\tau} n_1, \quad (1)
\]
\[
\frac{\partial W_p}{\partial t} = D \nabla^2 W_p - \sigma_a c n_0 W_p + \frac{1}{\tau} I_{in}, \quad (2)
\]
\[
\frac{\partial n_1}{\partial t} = \sigma_a c n_0 W_p - (\sigma_e c n_1 - \sigma_r c n_0) W - \frac{1}{\tau} n_1. \quad (3)
\]

Here \( D \) is the diffusion constant; \( \sigma_e, \sigma_r, \) and \( \sigma_a \) are the emission, reabsorption, and absorption cross sections of the dye molecules, respectively; \( \tau \) is the spontaneous emission lifetime; and \( n_0 \) is the density of molecules in the ground state. The source term for the pump light, \( t^{-1} I_{in}(z,t) \), is a pulse with width \( \tau_p \) and decays exponentially in \( z \) with an extinction length \( \ell_{ext} = \ell_a^{-1} + t^{-1} \) (with \( \ell_a \) as the absorption length). In our system, scattering is responsible for most of the extinction (\( t \ll \ell_a \)), and hence the source term is treated as power independent. The term \( \beta \) is the spontaneous emission factor and enables us to take into account spectral narrowing without making the equations explicitly spectrally dependent.

Partial differential Eqs. (1) and (2) are converted into ordinary differential equations in time by the method of lines. The set of equations is then solved numerically using MATLAB [12]. For that purpose, a nonuniform cylindrical grid is introduced to allow the calculation of a relatively large system in two dimensions and, at the same time, to have detailed information close to the origin [13]. Defining \( h_+ \equiv x_{i+1} - x_i \) and \( h_- \equiv x_i - x_{i-1} \), the finite difference form of the first and second derivatives read

\[
\frac{\partial f}{\partial x} \to \frac{f(x + h_+) - f(x - h_-)}{h_+ + h_-}, \quad (4)
\]
\[
\frac{\partial^2 f}{\partial x^2} \to \frac{h_- f(x + h_+) - (h_+ + h_-)f(x) + h_+ f(x - h_-)}{0.5h_+h_-(h_+ + h_-)}. \quad (5)
\]

The two differential equations describing the laser light and the pump light, Eqs. (1) and (2), obey mixed boundary conditions: the photon densities are assumed zero at a characteristic length scale (the extrapolation length) outside the sample. The sample is modeled as a cylinder with dimensions \( r = 150 \ell \) and \( z = 150 \ell \). The time taken into consideration spans 40 ns. Realistic experimental situations [11] \(( \ell = 5 \mu m, \quad \rho = n_1 + n_0 = 6 \times 10^{23} \text{m}^{-3}, \quad \sigma_a = 1.6 \times 10^{20} \text{m}^{-2}, \quad \sigma_r = 0.073 \times 10^{-26} \text{m}^4, \quad \sigma_e = 4 \times 10^{-20} \text{m}^4, \quad \beta = 0.07, \quad \tau_p = 5 \text{ ns}) \) were calculated on a \( 40 \times 50 \times 60 \) \(( r \times z \times r \times z \) \) grid.

In Fig. 3(b) we show the 50% contour of the numerically determined time-integrated diffusive emission photon density profile below and above threshold in the \( r-z \) plane. The profile above threshold is expanded compared to the profile below threshold. We compare experiment and theory by analyzing cuts from the numerical data at \( r = 30 \mu m \) in the \( r-z \) plane. In Fig. 3(c), the values for \( z \) at which the intensity profile reaches its maximum and the width of the intensity profile are plotted versus the pump power for both the numerical calculation and the experiment. In agreement with experimental observations, the spatial width of the laser light profile increases around threshold but remains constant far above threshold. The inclusion of reabsorption is important in order to make the results quantitatively correct.

To understand why the spatial profile is fixed far below and far above threshold, we analyze the contribution of spontaneous and stimulated emission to the total emission in time. Figure 4 shows calculated contours of the population of the upper laser level for pump powers below and above threshold in the \( z-t \) plane. Far below threshold, the origin of the emitted light is spontaneous emission. In this regime, the set of equations is linear; an increase in the pump fluence simply leads to more spontaneous emission events everywhere in the sample, but does not change the normalized spatiotemporal profile of the excited molecules.

Above threshold, stimulated emission is responsible for most (>99%) of \( P/P_{th} > 1 \), where \( P_{th} \) is the pump power threshold) of the light emitted by the dye molecules. Gain compensates for the losses in the system, but the gain can never, except for temporal fluctuations, actually exceed the losses. The main losses in a random laser are due to the diffusive loss term, \( \nabla^2 W \), which is
only nonzero close to the origin. As a consequence, the gain length in a random laser is strongly spatially dependent. Once above threshold, the stronger the system is pumped, the deeper inside the sample reabsorption losses are compensated for. The cancellation of reabsorption hardly changes the diffusive laser light profile, since the required gain is small compared to the gain needed to compensate the diffusive losses. Analogous to conventional laser systems the gain saturates above threshold. Around the random laser threshold, the expansion of the lasing volume rather than a temporal maximum serves. The expansion of the lasing volume rather than a decrease in gain length is an important factor not yet taken into account for explaining the observation of an increasing number of spikes with increasing pump power [16].

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References